Kerbin-Duna Aldrin Cycler Trajectory

Christopher Hayes

Abstract

For the uninitiated, Kerbin and Duna are the Earth and Mars equivalent planets in the space flight game, Kerbal Space Program. This article contains the mathematical establishment of a Kerbin-Duna Aldrin Cycler trajectory plus instructions for how to establish such a trajectory in the game, and an overview of its uses. This guide assumes a basic understanding of method of patched conics orbital mechanics, and makes the simplifying assumption that Kerbin and Duna are both in circular orbits with no relative inclination. For the mathematical solution, we will also assume that the cycler never enters Duna's sphere of influence, or Hill Sphere, during the cycler's intercept of Duna.

This article somewhat follows along with "Cycler Orbit Between Earth and Mars", authored by Dennis V. Byrnes, James M. Longuski, and Buzz Aldrin.

Key concepts of orbital mechanics are linked to their respective Wikipedia entries. For simplicity, radii are given in astronomical units (AU), where $R_K = 1\ AU$.

The solutions have been found with a MatLab script which can be downloaded here, and pictures that follow along with this article and the script can be found here.

Introduction

A cycler trajectory is an orbit that meets up with 2 planets at regular time intervals. Originally theorized by Buzz Aldrin in 1985, the Aldrin Cycler is a 1-synodic period Earth-Mars cycler. The Aldrin Cycler is the shortest possible repeating Earth-Mars trajectory, return to Earth once every 2.135 years, where the cycler receives a gravity assist from Earth, altering its orbit to once again return to Mars.

Earth's gravity is not strong enough to fully establish the desired return trajectory to Mars with a gravity assist alone, thus a small propulsive maneuver must be performed for the cycle to continue.

There are two types of Aldrin Cycler orbits, "up" (outbound) and "down" (inbound); these refer to having a shorter trip to or from Mars, with the short trip being around 5 months and the long trip being around 20 months. We'll be creating an "up" cycler, however, the same math works for creating a "down" cycler with the only alteration being the position of Duna during launch.

A video overview of an Aldrin Cycler can be found at http://www.youtube.com/watch?v=qCVfUlFZQ4U.

Contents

Aldrin Cycler Trajectory

Establishing our Coordinate System

We will be using both Cartesian and radial coordinates to describe our system. We will consider the vernal equinox of our system to be along the x-axis ($\theta = 0^o$ in radial coordinates). At epoch (time $t = 0$), Kerbin will be at vernal equinox. The injection burn will occur at epoch.

Synodic Period and Shift

To establish the Aldrin Cycler trajectory, we must first calculate the synodic period of Kerbin and Duna using equation (1), we calculate $T_{Syn} \approx 2.13\ Kerbin\ Years$. This means that our cycler must return to Kerbin once every $2.13\ Kerbin\ Years$ if we want it to return to Duna on the same trajectory each cycle.

We also must calculate the relative angular shift, $\Delta\psi$, of Kerbin and Duna with each complete synodic period. Though Kerbin and Duna will return to the same relative alignment every $2.13\ Kerbin\ Years$, they will not be in the same position relative to the original launch of the cycler (i.e. relative to the vernal equinox). With equation (2) we calculate the angular shift of synodic alignment to be $\Delta\psi = 48.46^o$ This is distance that Kerbin advances over a period of $.13\ Kerbin\ Years$; its' shift from its position at the cycler launch. This shift is also known as the advance of the cycler perihelion.

Lambert's Problem

To define our orbit's shape, we need to calculate the semi major axis, a, and the eccentricity, e. To correctly calculate these parameters, we must solve the Multi-revolution Lambert Problem with our given boundary conditions. The Multi-revolution Lambert Problem is the set of non-trivial solutions to the equation $\ddot{\bar{r}}_C = -\mu_S \cdot \frac{\hat{r}_C}{r_C}$, given boundaries of initial position, $\bar{r}_o$, final position, $\bar{r}_f$, elapsed time between the two positions, t, and number of complete orbits during the elapsed time, m. For this, I have used the MatLab function "Robust Solver for Lambert's Orbital Values Problem" developed by Dr. D. Izzo of the European Space Agency Advanced Concepts Team. This function implements two solver algorithms, one developed by Dr. Izzo which converges quickly but often fails at large numbers of revolutions, and one developed by E. R. Lancaster and R. C. Blanchard, modified by R. H. Gooding, which is slower but solves much more robustly. A link to the solver can be found in the works cited.

As a note, there are 2 unique solutions to each set of inputs for the lambert problem, a "short way" and a "long way"; the "long way" will have a higher eccentricity and a larger semi major axis. The solver by default returns the "short way", which is the solution we are interested in.

Inputs

We must give our inputs in Cartesian coordinates. We set our initial position to r1 = $R_K[1\ , 0, 0]$ (Kerbin's position at vernal equinox), and our final position to same distance, shifted forward by angle $\Delta\psi$. In Cartesian coordinates we can thus define r2 = $R_K[cos\ \Delta\psi, \sin \Delta\psi, 0]$. Note that R_K must be in units of km for the solver to function correctly. Next we need the flight time in units of $Earth\ Days$. Simply, $t = T_{Syn} = 227.38\ Earth\ Days$. Next, we must find the number of complete orbits we want to occur in the elapsed time. We want to return to Kerbin $\Delta\psi$ ahead of its initial position; this means that we want to return the cycler to its own initial position and then advance by angle $\Delta\psi$, where we meet up with Kerbin once again. We want to complete $1 + \Delta\psi\ orbits$, thus $m = 1\ complete\ orbit$. Lastly, we need the gravitational parameter, μ, of Kerbol (Sun) in $^{km^3}/_{s^2}$.

Outputs

The solver returns 3 vectors, V1 and V2, the velocities at the given points r1 and r2, and a vector giving the ApR and PeR, the apoapsis and periapsis as distances from the gravitational center (not from the surfaces often used in KSP!). These values are returned as $2.22AU$ and $.975\ AU$. With these values, we determine orbital elements $a = 1.60\ AU$ and $e = .390$ from equations (3) and (4).

For our flight, we will need to know apoapsis and periapsis as distances from Kerbol's surface; which we find trivially with equations (5) and (6).

Constraining the Orbit

To fully define our orbital path, we must determine its angular alignment in the plane it is constrained to. We know the periapsis of the cycler orbit is half-way between the two intersections with Kerbin's orbit, thus the Longitude of the Periapsis, $LPe\ = \frac{\Delta\psi}{2} = 24.2^o$. This also means that cycler's true anomaly at epoch is $\nu_o = -24.2^o$ relative to the cycler's periapsis (NOT relative to the vernal coordinate system!), which we will use later on.

We have now successfully defined the orbit in our coordinate system with the 3 parameters, $a = 1.60\ AU$, $e = .390$, and $LPe\ = 24.2^o$.

Duna Intercept

To successfully intercept Duna, we must find the phase angle between Kerbin and Duna at launch. To do this we must first determine the position in Duna's orbit where the cycler will intercept the planet, θ_{D_I}, Duna's true anomaly at intercept. This can be calculated from equation (7) when we rearrange it in to equation (8). Note that we must subtract the initial true anomaly of the cycler, ν_o, from the result of the inverse cosine function to fit our coordinate system where the x-axis is the vernal equinox. This gives a value of $\theta_{D_I} = 130.6^o$.

Next, to find Duna's initial position, θ_{D_o}, we must find the time elapsed in the first leg of the cycler's trip. To do this, we must define new parameters of the cycler orbit. We must find the Eccentric Anomaly, E, the Mean Anomaly, M, and the Mean Motion, n, as defined by equations (9)-(12). When solving for these values, we must remember that we are starting at an initial true anomaly of $\nu_o = -24.2\ deg$. We will find 2 parameters, E_I and E_o from equations (9) and (10), using the radius at Duna intercept and the initial true anomaly. Plugging in to equations (11) and (12), we find $\Delta M = 1.24\ rad$ and $n = 3.38 * 10^{-7} rad/s$. From these, we can rearrange equation (13) to solve for elapsed time, $t = 42.4\ Earth\ Days\ (169.6\ Kerbin\ Days)$.

We have found out how long our flight to Duna will take, and we know Duna's true anomaly at intercept. Thus, since we know Duna's orbital period, we can easily find Duna's initial true anomaly by solving $\theta_{D_o} = \theta_{D_I} - \frac{t}{T_D}$. From this we find $\theta_{D_o} = 54.5^o$. Since we are measuring this angle from vernal equinox, and Kerbin is at the vernal equinox at launch, the phase angle between Kerbin and Duna is also $\varphi = \theta_{D_o} = 54.5^o$.

Cycler Orbit Injection

The solver has given us two other output vectors; our velocities at points r1 and r2. We want our hyperbolic excess velocity at point r1 to match the value returned by the solver. For simplicity, we will start from a circular parking orbit at 100 km above Kerbin's surface, where $R_{CK_o} = 700\ km$. Our speed at point r1, equivalent to our hyperbolic excess velocity, $v_\infty = 1.97\ km/s$. We can find using equation (14) that our velocity in our parking orbit is $v_{C_o} = 2.25\ km/s$. Using equations (15)-(18), we find we need a velocity change of $\Delta v = 1491\ m/s$. This burn will be entirely in the prograde direction.

Now that we know our burn magnitude, we must find the position of our burn. We will define this as an angle to Kerbin's prograde velocity vector, its instantaneous direction of motion. This is called the ejection angle, α. Using the diagram ****** below, we see geometrically that $\alpha = \frac{3}{2}\pi - (\beta + \gamma)$, knowing that the internal angles in a triangle sum to π. β and γ are defined in equations (19) and (20), and solving, we find that, $\alpha = 84.5^o$ to Kerbin prograde.

We now know everything we need to establish the cycler's orbit – at a phase angle of $\varphi = 54.5^o$, we perform a prograde burn of $\Delta v = 1491\ m/s$, with an ejection angle of $\alpha = 84.5^o$ to Kerbin prograde. To continue the cycle, however, we must calculate a gravity assist and burn at Kerbin.

Kerbin Flyby

We will use a Kerbin flyby to impart a velocity change to our vehicle, causing our trajectory about Kerbol to change. We will also determine the required propulsive maneuver to match our flyby trajectory to original injection trajectory. For simplicity, we will be using a new (prime) reference frame to calculate the flyby. We will assume the vernal equinox to be Kerbin's position during the flyby ($\theta = 48.5^o$ in the original reference frame).

To perform the Kerbin flyby, we must choose our desired periapsis at Kerbin. Since we want to mimic our injection, and we know that our injection burn was performed at an angle to prograde of $\alpha = 84.5^o$ and $R_{C_o} = 700km$, we want to choose a periapsis that will cause our flyby trajectory to intersect with this point. To do this, we must iterate. Following through with iteration of equations ()-(), we eventually find that a trajectory with a periapsis of $R_C = 692\ km$ will intersect this point.

Propulsive Maneuver

Found experimentally, the burn is -27.2 m/s prograde and -453.8 m/s radial (total burn of 454.6 m/s at -86 deg pitch and 90 deg heading). The following segment will be completed after the last few lines of code are debugged.

We know the location our burn will occur at. We need to define the magnitude and direction of our burn. We know that our desired velocity is [Co_v+Cinj_delta_v, 0] (same as our injection condition in the original frame). We must then calculate our initial velocity at the point, and find the difference. Equations ()-() give us an initial velocity of […]. Subtracting from our known final velocity of […], we find we must perform a burn of […] km/s, with the x value being in the prograde direction, and the y value being in the radial direction.

Appendix A: How to use The Aldrin Cycler

A cycler is a convenient way to repeatedly transfer people and cargo between 2 planets. If a self-sustaining cycler station were to exist, a small transport ship could bring people and cargo to and from the station, with the station providing all the necessary amenities, radiation shielding, power, etc., for the duration of the trip, as can be very spacious. For those using the KSP mod TAC life support, an Aldrin Cycler can be a great tool for interplanetary colonization. It is important to keep in mind that any ship going to the cycler must match its velocity, so small transport/supply ships are best used for this purpose.

Mission Plan 1

This is a repeated 1-way mission to Duna. We will use 2 craft types for this, the cycler (a), and multiple Duna landers (b). For relevant pictures, see here.

1. Establish $100km$ parking orbit (a&b1, docked)
2. Wait for Kerbin Duna Phase alignment of $\varphi = 54.5^o$
3. Perform prograde burn of $1491\ km/s$ at ejection angle $\alpha = 84.5^o$
4. Perform corrections to achieve Duna intercept
5. Once outside of Kerbin SOI, adjust so $ApA = 29.979\ Gm$ and $PeA = 13.002\ Gm$
6. Once Kerbin closest approach is no longer shown as current position on the map (after cycler periapsis?), adjust trajectory so that Kerbin intercept achieved at the second crossing point of the cycler and Kerbin
7. Eject(b1) a few days before Duna intercept, then land (b1) on Duna
8. Before (a) Kerbin flyby, adjust (a) trajectory so that periapsis at Kerbin is 92km on the dark (night)side, and launch (b2) to a $100km$ parking orbit
9. Less than an hour before (a) Kerbin flyby, inject (b2) on to approximate cycler trajectory, but slow by $\sim 100km/s$ ($1391\ m/s$ injection burn instead of $1491m/s$)
10. Perform flyby correction burn of -27.2 m/s prograde and -453.8 m/s radial (total burn of 454.6 m/s at -86 deg pitch and 90 deg heading on navball) of with (a) at ejection angle $\alpha = 84.5^o$.
11. Adjust (b2) trajectory to intercept (a)
12. Dock (b2) to (a)
13. Repeat from step 4.

Alternate Mission

A “down” cycler can also be established with a recalculation of Duna’s phase angle at injection. The “down” cycler can run return missions from Duna. For this, we use 4 craft types for this, up and down cyclers (au) and (ad), Kerbin-cycler ferries (b) and a Duna-cycler ferries (c). This can be initiated after many Kerbals have been brought to Duna via Mission Plan 1, or another method, to get some of them back home.

Appendix B: List of Variables

R : Orbit Radius

r: Planet Radius

T: Period

T_{Syn}: Synodic Period

$\Delta\psi$: Advance of perihelion per orbit

a : Semi-Major Axis

e : Eccentricity

μ : Gravitational Parameter

PeR : Periapsis (from gravitational center)

ApR : Apoapsis (from gravitational center)

PeA : Periapsis (from surface)

ApA : Apoapsis (from Surface)

LPe : Longitude of the Periapsis

ν : Cycler True Anomaly

θ : Planet True Anomaly

γ : Cycler Flight Path Angle at Kerbin

δ : Half Turn Angle

v_C : Cycler Orbital Velocity at Kerbin

Δv_C : Cycler Velocity Change at Kerbin

t : Time

φ : Kerbin-Duna Launch Phase Angle (relative to Kerbin)

E : Eccentric Anomaly

Variable Subscripts

K: Kerbin

D: Duna

C: Cycler

S: Kerbol (Sun)

o : Condition at Epoch

I : Condition at time of Duna Intercept

Appendix C: Equations

$$T_{Syn} = \frac{T_K T_D}{T_K - T_D} \quad (1)$$

$$\Delta\psi = \left[decimal\ portion\ of\ \frac{T_{Syn}}{T_K}\right] * (2\pi\ rads) \quad (2)$$

$$a = ApR + PeR \quad (3)$$

$$e = \frac{ApR - PeR}{a} \quad (4)$$

$$ApA = ApR - \mathrm{r}_S \quad (5)$$

$$PeA = PeR - \mathrm{r}_S \quad (6)$$

$$R = \frac{a(1-e^2)}{(1+ecos(\theta-\nu_o))} \quad (7)$$

$$\theta_{D_I} = \nu_o + \cos^{-1}\left[\frac{1}{e}\left(\frac{a}{R_D}(1-e^2)-1\right)\right] \quad (8)$$

$$E = \cos^{-1}\left[\frac{1}{e}\left(1-\frac{R_D}{a}\right)\right] \quad (9)$$

$$E = \cos^{-1}\left[\frac{e+\cos\nu}{1+e\cos\nu}\right] \quad (10)$$

$$M = E - e\sin E \quad (11)$$

$$n = \sqrt{\frac{\mu}{a^3}} \quad (12)$$

$$M = nt \quad (13)$$

………

$$\tan\gamma = \frac{e_C R_K}{a_C(1-e_C{}^2)} \quad (7)$$

$$v_{C_K} = \sqrt{\frac{\mu_S}{R_K}\left(2-\frac{R_K}{a_C}\right)} \quad (8)$$

$$\Delta\, v_{C_K} = 2\, v_{C_K}\sin\gamma \quad (9)$$

$$v_{\infty_{C_K}} = \sqrt{v_C{}^2 + v_K{}^2 - 2v_C v_K\cos\gamma} \quad (10)$$

$$\sin\delta = \frac{\Delta v_C}{2v_\infty} \quad (11)$$

$$R_{P_{C_K}} = \frac{\mu_K}{v_{\infty_{C_K}}{}^2}(\csc\delta - 1) \quad (12)$$

$$\mathrm{R}_C = R_K \frac{1+e_C \cos\theta_{C_K}}{1+e_C \cos\theta} \tag{13}$$

$$v_{C_o} = \sqrt{\frac{\mu_K}{R_{C_o}}} \tag{14}$$

Appendix D: Constants and Relevant Calculated Values

Kerbin

$\mathrm{T}_K = 9.2035\ Ms$

$\mathrm{R}_K = 13.600\ Gm = 1\ AU$

$\mu_K = 3.5316 * 10^{12}\ \frac{m^3}{s^2}$

$v_K = 9284.5\ m/s$

$\mathrm{r}_{K_{Atmosphere}} = 670\ km$

$SOI_K = 84.159\ Mm$

Duna

$\mathrm{T}_D = 17.315\ Ms$

$\mathrm{R}_D = 20.726\ Gm = 1.52\ AU$

Kerbol (Sun)

$\mu_S = 1.1723 * 10^{18}\ \frac{m^3}{s^2}$

$\mathrm{r}_S = 261.6\ Mm$

Kerbin-Duna Calculations

$T_{Syn} = 19.646\ Ms = 2.1357\ Kerbin\ Years$

$\frac{\Delta\psi}{2} = .42279\ rad = 24.224^o$

Cycler Orbit

$a_C = 21.752\ Gm$

$e_C = .39023$

$ApA = 29.979\ Gm$

$PeA = 13.002\ Gm$

$LPe = .42279\ rad = 24.224^o$

$\nu_o = -LPe$

Duna Intercept

$\theta_{D_I} = 2.2802\ rad = 130.65^o$

$\Delta E = 1.1673\ rad = 66.880^o$

$\Delta M = 1.2359\ rad$

$n = 3.3750 * 10^{-7}\ rad/s$

$t = 2.3952\ Ms = 27.723\ Earth\ Days$

$\varphi = \theta_{D_o} = .9514\ rad = 84.512^o$

Cycler at Kerbin

$\gamma = .2788\ rad = 15.97^o$

$v_{C_K} = 10890\ m/s$

$\Delta\ v_{C_K} = 5994\ m/s$

$v_{\infty_{C_K}} = 3223\ m/s$

$2\delta = 2.388\ rad = 136.82^o$

$\Delta v_g =$

$R_P = 700\ km$

$PeA = -171.26\ km$

Sources

http://www.mathworks.com/matlabcentral/fileexchange/26348-robust-solver-for-lambert-s-orbital-boundary-value-problem

Curtis, Howard D. Orbital Mechanics for Engineering Students 2nd ed. ISBN : 978-0-12-374778-5

http://www.braeunig.us/space/orbmech.htm (zenith angle)

https://engineering.purdue.edu/people/james.m.longuski.1/JournalArticles/1993/CyclerOrbitbetweenEarthandMars.pdf